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Research article

HYDROMAGNETIC GRAVITATIONAL STABILITY OF FLUID CYLINDER PENETRATED INTERNALLY BY AZIMUTHAL VARYING MAGNETIC FIELD

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Abstract

The self-gravitating instability of fluid cylinder penetrated by azimuthally varying magnetic field internally has been developed. Upon using the linear perturbation technique, the problem is studied, the dispersion relation is established and discussed. Some reported works are recovered from the present general data as limiting cases with suitable simplifications.

Keywords: Hydromagnetic, Self-gravitating, Azimuthal, Stabili

1- Introduction

The self-gravitating instability of a fluid cylinder has been studied for first time by Chandrasekhar and Fermi (1953). See also Oganesian (1956). Chandrasekhar (1981) documented the results studies. Radwan (1989) and (1991) has studied the pure self gravitating instability of a fluid cylinder surrounded by another fluid of different density whether the latter is radially bounded or not. Such kind of studies are very important not only from the academic view-point but also for their crucial applications of many phenomena ranging from laboratory scale to astrophysical ones e.g. stability of spiral arm of galaxy,…etc., where the latter idea has received support from the arguments of a quasi-stationary spiral structure, cf. Lin, (1966), Kakutani et al, (1994), Lardner (1983) and Radwan (2005). The stability of different cylindrical models under the action of self gravitating force in addition to other forces has been elaborated by Radwan and Hasan (2008) and (2009). They (2008) studied the gravitational stability of a fluid cylinder under transverse time-dependent electric field for axisymmetric perturbations. Hasan (2011) has discussed the stability of oscillating streaming fluid cylinder subject to combined effect of the capillary, self gravitating and electrodynamic forces for all axisymmetric and non axisymmetric perturbation modes. Hasan (2011) studied the instability of a full fluid cylinder surrounded by

self-gravitating tenuous medium pervaded by transverse varying electric field under the combined effect of the capillary, self-gravitating and electric forces for all modes of perturbations. He (2012) discussed the instability of a full fluid cylinder surrounded by selfgravitating tenuous medium pervaded by transverse varying electric field under the combined effect of the capillary, self-gravitating and electric forces for all modes of perturbations. He (2012) studied the magnetodynamic stability of a fluid jet pervaded by transverse varying magnetic field while its surrounding tenuous medium is penetrated by uniform magnetic field.
Here we investigate the axisymmetric hydromagnetic instability of a fluid cylinder penetrated by azimuthal varying magnetic field.

2- Formulation of the Problem

Consider a fluid cylinder (radius R_o) surrounded by tenuous medium of negligible motion. The fluid is assumed to be non-viscous, incompressible and perfectly conducting and penetrated by the toroidal varying magnetic field

$$\underline{H}_0 = (0, H_0 r, 0) \tag{1}$$

The tenuous medium is pervaded by the axial magnetic field

$$\underline{H}_0^{tn} = (0, 0, \alpha H_0) \tag{2}$$

where H_o is the intensity of the magnetic field in the tenuous medium as the parameter α (=1). The components of $\underline{H}_0$ and $\underline{H}_0^{tn}$ are considered along the cylindrical coordinates (r, φ ,z) with the z-axis coinciding with the axis of the fluid cylinder. The model is acting upon the self gravitating, electromagnetic and the fluid pressure gradient forces. The basic equations are coming out from the combination of Maxwell equations concerning the electromagnetic theory, the ordinary fluid dynamic equations and Newtonian self-gravitating equations. For the problem at hand, these equations may be given as follows.
In the fluid:

$$\rho \frac{d\underline{u}}{dt} = -\nabla P + \mu(\nabla \wedge \underline{H}) \wedge \underline{H} + \rho \nabla V \tag{3}$$

$$\frac{\partial \underline{H}}{\partial t} = \nabla \wedge (\underline{u} \wedge \underline{H}) \tag{4}$$

$$\nabla \cdot \underline{u} = 0 \tag{5}$$

$$\nabla \cdot \underline{H} = 0 \tag{6}$$

$$\nabla^2 V = -4\pi G \rho \tag{7}$$

with

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \underline{u} \cdot \nabla \quad , \quad \nabla^2 = \nabla \cdot \nabla \tag{8}$$

$$\nabla = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \varphi}, \frac{\partial}{\partial z} \right) \tag{9}$$

In the tenuous medium

$$\nabla \cdot \underline{H}^{tn} = 0 \tag{10}$$

$$\nabla \wedge \underline{H}^{tn} = 0 \tag{11}$$

$$\nabla^2 V^{tn} = 0 \tag{12}$$

Here ρ, $\underline{u}$ and P are the fluid mass density, velocity vector and kinetic pressure, μ and $\underline{H}$ are the magnetic field permeability coefficient and intensity, G and V are the self gravitating constant and potential. Equation (3) is MHD gravitational vector equation of motion, equation (4) is the elevation equation of magnetic field valid for non-resistive magnetized fluid, equation (5) is the continuity equation concerning incompressible fluid, equation (6) is the conservation of magnetic flux and idem equation (10). Equations (7) and (2) are the Newtonian self gravitating equations in the fluid and tenuous medium while equation (11) is Maxwell equation where there is no current in the tenuous medium.

3 - Unperturbed State

The fundamental equations (3) – (12) in the equilibrium state are solved with cylindrical and longitudinal symmetries $\frac{\partial}{\partial \varphi} = 0$ and $\frac{\partial}{\partial z} = 0$. The solution is matched across the boundary surface at $r=R_o$.

The self gravitating potentials in the fluid V_0 and in the tenuous medium V_0^{tn} are given by:

$$V_0 = -\pi G \rho r^2 \tag{13}$$

$$V_0^{tn} = -2\pi G \rho R_0^2 \left[\ln(r / R_0 \right] + C_1 \tag{14}$$

where C_1 is an additive constant while the quantities with subscripts 0 indicate their values in the unperturbed state, taking into account for the problem at hand that the unperturbed state is a static one $\underline{u}_0 = 0$. The hydromagnetic self gravitational equation of motion (3), yields

$$\nabla \Pi_0 = 0 \tag{15}$$

with

$$\Pi_0 = C_2 \tag{16}$$

Such that

$$\Pi_0 = \left(\frac{\mu}{2}\right) H_0^2 r^2 + (P_0 / \rho) - V_0 \tag{17}$$

The constant of integration C_2 could be determined upon applying the boundary condition that the balance of the pressure must be satisfied across the boundary $r= R_0$. Consequently, the distribution of the fluid pressure P_0 in the unperturbed state is given by

$$P_0 = \pi G \rho^2 \left(R_0^2 - r^2\right) + (\mu / 2) H_0^2 (\alpha^2 - r^2) \tag{18}$$

It is worthwhile to mention here that P_o must be non-negative across the interface $r = R_o$ to insure the existing of the unperturbed state. Therefore, in order that

$$P_0 \geq 0 \tag{19}$$

at $r = R_o$, the parameter α must satisfy the condition

$$\alpha \geq R_0 \tag{20}$$

where the equality holds as a limiting case with zero fluid pressure.

4 - Linearization

Consider an axisymmetric sinusoidal wave along the fluid cylinder interface. For a small departure from the unperturbed state, linearization of the basic equations (3) – (12) is accomplished by substituting the expansions

$$P(r, z, t) = P_0 + \varepsilon(t)\, P_1 (r,z) \tag{21}$$

$$\underline{u}(r, z, t) = \underline{u}_0 + \varepsilon(t)\, \underline{u}_1 (r,z) \tag{22}$$

$$\underline{H}(r, z, t) = \underline{H}_0 + \varepsilon(t)\, \underline{H}_1 (r,z) \tag{23}$$

$$V(r, z, t) = V_0 + \varepsilon(t)\, V_1 (r,z) \tag{24}$$

$$\underline{H}^{tn} (r, z, t) = \underline{H}_0^{tn} + \varepsilon(t)\, \underline{H}_1^{tn} (r,z) \tag{25}$$

$$V^{tn} (r, z, t) = V_0^{tn} + \varepsilon(t)\, V_1^{tn} (r,z) \tag{26}$$

and retaining only first-order terms in the small fluctuating variables P_1, $\underline{u}_1$, $\underline{H}_1$, V_1, $\underline{H}_1^{tn}$ and V_1^{tn}. Here $\varepsilon(t)$ is, the amplitude of the perturbation at instant t, being

$$\varepsilon(t) = \varepsilon_0 \exp (i\omega t) \tag{27}$$

where ε_o is the initial amplitude at t=0 while ω is the oscillation frequency of the perturbed wave.

The linearized perturbation equations are given as follows.

In the fluid:

$$\frac{\partial \underline{u}_1}{\partial t} - (\mu / \rho)(\underline{H}_1 . \nabla) H_0 - (\mu / \rho)(\underline{H}_0 \cdot \nabla) \underline{H}_1 = -\nabla \Pi_1 \tag{28}$$

$$\Pi_1 = (P_1 / \rho) - V_1 + (\mu / 2\rho)(2 \underline{H}_0 \cdot \underline{H}_1) \tag{29}$$

$$\nabla.\underline{u}_1 = 0 \quad , \quad \nabla.\underline{H}_1 = 0 \tag{30), (31}$$

$$\frac{\partial \underline{H}_1}{\partial t} = (\underline{H}_0.\nabla)\underline{u}_1 - (\underline{u}_1.\nabla)\underline{H}_0 \tag{32}$$

$$\nabla^2 V_1 = 0 \tag{33}$$

In the tenuous medium

$$\nabla.\underline{H}_1^{tn} = 0 \tag{34}$$

$$\nabla \wedge .\underline{H}_1^{tn} = 0 \tag{35}$$

$$\nabla^2 V_1^{tn} = 0 \tag{36}$$

Based on the linear perturbation technique, the perturbed radial distance of the fluid cylinder may be given by

$$R = R_0 + R_1 \quad , \quad R_1 << R_0 \tag{37}$$

with

$$R_1 = \varepsilon(t) \exp i(kz) \tag{38}$$

Here R_1 is the elevation of the surface wave measured from the unperturbed position, where k (real number) is the longitudinal wave number.

In view of the time-space dependence (38), the linearized system of equations (28) – (33) could be expressed in terms of the different vectors in the following form

$$i\omega u_{1r} = \frac{\partial \Pi_1}{\partial r} \tag{39}$$

$$u_{1\varphi} = 0 \tag{40}$$

$$i\omega u_{1z} = ik\,\Pi_1 \tag{41}$$

$$\frac{\partial^2 u_{1r}}{\partial r^2} + \frac{1}{r}\frac{\partial u_{1r}}{\partial r} + \frac{\partial u_{1z}}{\partial z} = 0 \tag{42}$$

$$\frac{\partial^2 H_{1r}}{\partial r^2} + \frac{1}{r}\frac{\partial H_{1r}}{\partial r} + \frac{\partial H_{1z}}{\partial z} = 0 \tag{43}$$

$$H_{1r} = 0 \tag{44}$$

$$i\omega H_{1\varphi} = -H_0 u_{1r} \tag{45}$$

$$H_{1z} = 0 \tag{46}$$

$$\frac{\partial^2 V_1}{\partial r^2} + \frac{1}{r}\frac{\partial V_1}{\partial r} - \frac{\partial^2 V_1}{\partial z} = 0 \tag{47}$$

Based on the linear perturbation technique, every perturbed quantity F_1 (r, z , t) could be expressed as

$$F_1(r,z,t) = F_1(r)\exp[\, i\,(kz + \omega t)] \tag{48}$$

where F_1 stands for u_{1r}, $u_{1\varphi}$, u_{1z}, H_{1r}, $H_{1\varphi}$, H_{1z} and Π_1.

By substituting from equations (39) – (41) into equation (42) and utilizing the expansion (48), we get

$$\frac{d^2 \Pi_1}{dr^2} + \frac{1}{r}\frac{d\Pi_1}{dr} - k^2\,\Pi_1 = 0 \tag{49}$$

The linearized perturbation equations in the tenuous medium are given by equations (34) – (36).

Equation (35) means, by the aid of the vector analysis theory, that $\underline{H}_1^{tn}$ may be derived from a scalar function Ψ_1 say, such that

$$\underline{H}_1^{tn} = -\nabla\Psi_1 \tag{50}$$

By using equation (3.50) for equation (3.34), we have

$$\nabla^2 \Psi_1 = 0 \tag{51}$$

Substituting the expansion (48) into equations (36) and (51), we get

$$\frac{d^2V_1^{tn}}{dr^2}+\frac{1}{r}\frac{dV_1^{tn}}{dr}-k^2V_1^{tn}=0 \qquad (52)$$

$$\frac{d^2\Psi_1}{dr^2}+\frac{1}{r}\frac{d\Psi_1}{dr}-k^2\Psi_1=0 \qquad (53)$$

Equations (49), (52) and (53) are in the form of Bessel equation, their solutions are given in terms of the modified Bessel functions.

Under the present circumstances, for the problem at hand, the non-singular solutions of equations (47), (49), (52) and (53) are given by: (cf. Gradshtyen and Ryzhik 1980)

$$\Pi_1\,(r,z,t) = AI_0\,(kr)\exp\,(i\,(\omega t + kz)) \qquad (54)$$

$$V_1\,(r,z,t) = BI_0\,(kr)\exp\,(i\,(\omega t + kz)) \qquad (55)$$

$$V_1^{tn}\,(r,z,t) = Ck_0\,(kr)\exp\,(i\,(\omega t + kz)) \qquad (56)$$

$$\underline{H}_1^{tn}\,(r,z,t) =-D\nabla K_0\,(kr)\exp\,(i\,(\omega t + kz)) \qquad (57)$$

Here, A, B, C and D are constants of integration to be determined while $I_0(kr)$ and $K_0(kr)$ are the modified Bessel functions of first and second kind of order zero.

5 - Dispersion Relation

In order to identify the constants of integration A, B, C and D, the solutions (54)-(57) must satisfy appropriate boundary conditions across the fluid interface at $r = R_o$. We have also to consider the contribution of the solutions (13), (14) and (18) due to perturbation and that the boundary surface $r = R_o$ is displaced according to the deformation (37).

For the problem at hand these boundary conditions are given as follows.

(i) The self gravitating potential $V(=V_0 + \varepsilon V_1)$ and its derivative must be continuous across the perturbed fluid interface at $r = R_o$, i.e.

$$V_1 + R_1\frac{\partial V_0}{\partial r} = V_1^{tn} + R_1\frac{\partial V_0^{tn}}{\partial r} \qquad (58)$$

$$\frac{\partial V_1}{\partial r} + R_1\frac{\partial^2 V_0}{\partial r^2} = \frac{\partial V_1^{tn}}{\partial r} + R_1\frac{\partial^2 V_0^{tn}}{\partial r^2} \qquad (59)$$

By substituting about V_0, V_1, R_1, V_1^{tn}, we get

$$B = 4\pi G\rho R_0 K_0(x) \qquad (60)$$

$$C = 4\pi G\rho R_0 I_0(x) \qquad (61)$$

(ii) The normal component of the magnetic field ($\underline{N}.\underline{H}$) must be continuous across the perturbed fluid interface at $r = R_0$. This condition reads

$$\underline{N}_0.\underline{H}_1 + \underline{N}_1.\underline{H}_0 = (\underline{N}_0.\underline{H}_1 + \underline{N}_1.\underline{H}_0)^{tn} \qquad (62)$$

where $\underline{N}$ is a unit outward normal vector to the perturbed fluid interface. It is given by

$$\underline{N} = N_0 + \varepsilon\underline{N}_1 \qquad (63)$$

with

$$\underline{N}_0 = (1,0,0) \qquad (64)$$

$$\underline{N}_1 = (0,0,-ik)R_1 \qquad (65)$$

By substituting about $\underline{N}_0$, $\underline{N}_1$, $\underline{H}_0$, $\underline{H}_1$, $\underline{H}_1$, $\underline{H}_1^{tn}$ and $\underline{H}_1^{tn}$ in the condition (62), yields

$$D = \frac{iH_0\alpha}{K_0'(x)} \qquad (66)$$

where $x(=kR_0)$ is the dimensionless longitudinal wave number.

(iii) The normal component of the velocity $\underline{u}$ must be compatible with the velocity of the fluid-tenuous perturbed fluid interface at $r = R_0$. This condition may be written as

$$\underline{N}_0.\underline{u}_1 + \underline{N}_1.\underline{u}_0 = \frac{\partial r}{\partial t} \tag{67}$$

Substituting for $\underline{N}_0, \underline{N}_1$, $\underline{u}_0$ (=0), $\underline{u}_1$ and r into the condition (3.67), we get

$$A = \omega^2 R_0 /(x I_0'(x) \tag{68}$$

(iv) Now, we have to go one more step after we have obtained all the problem variables in the unperturbed and perturbed states that we have to apply some compatibility condition.

"The normal component of the total stress tensor must be continuous across the perturbed fluid interface at r = R_o" . Mathematically this condition reads :

$$(\mu/2)\left[(2\underline{H}_0 \cdot \underline{H}_1) + R_1 \frac{\partial}{\partial r}(\underline{H}_0 \cdot \underline{H}_0)\right] + P_1 + R_1 \frac{\partial P_0}{\partial r} = \frac{\mu}{2}\left[(2\underline{H}_0.\underline{H}_1)^{tn} + R_1 \frac{\partial}{\partial r}(\underline{H}_0.\underline{H}_0)^{tn}\right] \tag{69}$$

By substituting for $\underline{H}_0$, $\underline{H}_0^{tn}$, $\underline{H}_1$ and $\underline{H}_1^{tn}$, R_1, P_1 and P_0 in the condition (69), we get the dispersion relation

$$\omega^2 = 4\pi G\rho \frac{x I_0'(x)}{I_0(x)}\left(\frac{1}{2} - I_0(x)K_0(x)\right) - \frac{\mu H_0^2}{\rho R_0^2}\frac{\alpha^2 x^2 I_0'(x)K_0(x)}{K_0'(x)I_0(x)} \tag{70}$$

6 - Discussions

The relation (70) is the magneto-gravitational stability criterion of self gravitating fluid cylinder pervaded by azimuthal magnetic field and surrounded by magnetized self gravitating tenuous medium penetrated by uniform magnetic field. The relation (70) is a linear combination of dispersion relations of a fluid cylinder subject to the electromagnetic force only and that one subject to self gravitating force only. It relates the oscillation frequency of the perturbed wave (along the fluid interface) ω with the modified Bessel functions $I_0(x)$, $K_0(x)$ of zero order and their derivatives and with the parameters G, ρ, μ, H_o and R_o, of the problem.

It is worthwhile to mention here that in absence of the magnetic field influence, the relation (70) reduces to

$$\omega^2 = 4\pi G\rho \frac{x I'(x)}{I_0(x)}\left(\frac{1}{2} - I_0(x)K_0(x)\right) \tag{71}$$

where use has been made of the recurrence relation

$$I_0'(x) = I_1(x) \tag{72}$$

The relation (71) is derived for first time by Chandrasekhar and Fermi (1953).

As we neglect the effect of the self gravitating force, the relation (70), yields

$$\omega^2 = \frac{\mu H_0^2}{\rho R_0^2}\frac{\alpha^2 x^2 I_1(x)K_0(x)}{K_1(x)I_0(x)} \tag{73}$$

where use has been made of the recurrence relations (3.72) and

$$K_0'(x) = -K_1(x) \tag{74}$$

6.1- Self Gravitating Stability

In such a case the cylindrical fluid cylinder is considered to be acted upon the self gravitating and the fluid pressure gradient forces only while the electromagnetic force influence is neglected. The dispersion relation of such a case is given by (71).

The discussion of this relation reveal that the model is self gravitating stable for short wavelengths while it is unstable for very long wavelengths. Numerically (see Abramowitz and Stegun (1970)) it is found that

$$\frac{\sigma^2}{4\pi G\rho} = \begin{cases} < 0, & as \quad 1.0667 < x < \infty \\ = 0, & as \quad x = 1.0667 \\ > 0, & as \quad 0 < x < 1.0667 \end{cases} \tag{75}$$

This means that the model is self gravitating unstable in the domain $0< x <1.0667$ while it is stable in the wide domain $1.0667 \leq x < \infty$ where the equality corresponds to the marginal stability state.

6.2 - Magento-Dynamic Stability

In this case the model is acting upon the electromagnetic forces with toroidal varying magnetic field interior the fluid jet and uniform magnetic field in the tenuous medium surrounding the jet. The dispersion relation of the present case is given by equation (73). The discussion of this relation reveal that the model is magnetodynamic stable for all values of $x \neq 0$ i.e. for all short and long wavelengths. It is clear that the uniform magnetic field pervaded in the tenuous medium has no influence on the stability of the fluid cylinder.
The stabilizing character of the azimuthal magnetic field in the fluid region due to the presence of the electromagnetic force $\mu(\nabla \wedge \underline{H}) \wedge \underline{H}$ may be interpreted as follows. This force is interpreted as arising from the action on the fluid of the Maxwell's stresses: a magnetic tension $\mu(\underline{H} \cdot \underline{H})/2$ per unit area along the magnetic lines of force and equal magnetic pressure acting in all directions in the conducting fluid. It is worthwhile to mention here that the latter is not perpendicular to the magnetic lines of force and acting in all directions because the diffusion term is neglected in the evolution equation of the magnetic field (4). Due to these stresses the lines of force are able to endow the fluid with a sort of rigidity

6.3 - Magneto-Gravitational Instability

This is the general case in which the fluid cylinder is acting upon the combined effect of the fluid pressure gradient, self-gravitating and electromagnetic forces. The dispersion relation of such case is given by equation (70).
The discussion of this relation could be carried out by the aid of the results of the subsections (6.1) and (6.2). The model is purely magneto-gravitational stable in the wide range $1.0667 \leq x < \infty$. While in the small range $0 < x < 1.0667$, the magnetic field decreases the destabilizing effect of the self gravitating force and simultaneously increases the stable domain. However, the magnetic field could not suppressed the self gravitating effect because the gravitational instability of sufficiently long waves will persist and the reason for this lies in the logarithmic singularity of the gravitational potential energy for infinite wavelengths.

In order to clarify such analytical discussions, the dispersion relation (70) has to be calculated numerically. For this aim the relation (70) may be written in the dimensionless form

$$\frac{\omega^2}{4\pi G\rho} = \frac{xI_1(x)}{I_0(x)}\left(\frac{1}{2} - I_0(x)K_0(x)\right) + \left(\frac{H_0}{H_G}\right)^2\left[\alpha^2 \frac{x^2 K_0(x)I_1(x)}{I_0(x)K_1(x)}\right] \qquad (76)$$

This relation is calculated for different values of α and (H_0/H_G). The data are tabulated and presented graphically where we find that the analytical results are confirmed numerically, see for example fig. (1) .

8 - Numerical Analysis

The dispersion relation (70) has been discussed numerically for all short and long wavelengths in which the dimensionless wave number is taken to be
$0 < x \leq 2$ and the corresponding values of σ or ω in the normal unit $\sqrt{4\pi\rho G}$ where ($\omega/2\pi$ is the frequency of oscillation) are determined. This has been performed for various values of $(H_0/\ H_G)$. Then for every value of $(H_0/\ H_G)$, different values of α is considered where $H_G = \rho R_0\sqrt{(4\pi\rho G/\mu)}$.

The numerical data are collected in tables, see tables (1) — (5) and presented in graphs, see figures (1) — (5). There are many features of interest in these tables and figures.

Corresponding to ($(H_0/\ H_G)$ = 0.1, as α = 1,1.1,1.2,1.5 and 3; it is found that the unstable domains are $0 < x < 1.041$, $0 < x < 1.036$, $0 < x < 1.031$, $0 < x < 1.014$ and $0 < x < 0.912$, while the neighboring stable domains are given by $1.041 < x < \infty$, $1.036 < x < \infty$, $1.031 < x < \infty$, $1.014 < x < \infty$ and $0.912 < x < \infty$.

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The critical points at which the transition from stable states to those of instability are occurred at x_c = 1.041 , 1.036, 1.031 , 1.014 and 0.912 respectively. See figure (1) and table (1) .

Corresponding to $((H_0/\ H_G)$ = 0.5, as α = 1,1.1,1.2,1.5 and 3; it is found that the unstable domains are $0 < x < 0.784$, $0 < x < 0.756$, $0 < x < 0.73$, $0 < x < 0.66$ and $0 < x < 0.441$, while the neighboring stable domains are given by $0.784 < x < \infty$, $0.756 < x < \infty$, $0.73 < x < \infty$, $0.66 < x < \infty$ and $0.441 < x < \infty$. The critical points at which the transition from stable states to those of instability are occurred at x_c = 0.784 , 0.756, 0.73 , 0.66 and 0.441 respectively. See figure (2) and table (2) .

Corresponding to $((H_0/\ H_G)$ =1, as α = 1,1.1,1.2,1.5 and 3; it is found that the unstable domains are $0 < x < 0.5675$, $0 < x < 0.537$, $0 < x < 0.5095$, $0 < x < 0.441$ and $0 < x < 0.2615$, while the neighboring stable domains are given by $0.5675 < x < \infty$, $0.537 < x < \infty$, $0.5095 < x < \infty$, $0.441 < x < \infty$ and $0.2615 < x < \infty$. The critical points at which the transition from stable states to those of instability are occurred at x_c = 0.5675 , 0.537, 0.5095 , 0.441 and 0.2615 respectively. See figure (3) and table (3) .

Corresponding to $((H_0/\ H_G)$ = 3, as α = 1,1.1,1.2,1.5 and 3; it is found that the unstable domains are $0 < x < 0.2615$, $0 < x < 0.2415$, $0 < x < 0.2245$, $0 < x < 0.185$ and $0 < x < 0.0985$, while the neighboring stable domains are given by $0.2615 < x < \infty$, $0.2415 < x < \infty$, $0.2245 < x < \infty$, $0.185 < x < \infty$ and $0.0985 < x < \infty$. The critical points at which the transition from stable states to those of instability are occurred at x_c = 0.2615 , 0.2415, 0.2245 , 0.185 and 0.0985 respectively. See figure (4) and table (4) .

From the above results we note that as we increase the magnetic field the values of x_c decrease and the unstable regions tends to be canceled to confirm this fact we will take the following case

Corresponding to $((H_0/\ H_G)$ = 10, as α = 1,1.1,1.2,1.5 and 3; it is found that the unstable domains are near to be canceled and the values of the growth rate of oscillation are directly increase as the values of x increase. See figure (5) and table (5) .

9 - Conclusions

The self gravitating instability of fluid cylinder penetrated by toroidal varying magnetic field internally has been developed. Upon using the linear perturbation technique, the problem is studied, the dispersion relation is established and discussed. Some reported works are recovered from the present general data as limiting cases with suitable simplifications.. The electromagnetic force has stabilizing effect for all perturbed wavelengths. The uniform magnetic field penetrated in the tenuous medium has no direct influence on the stability of the model.
The self gravitating force is stabilizing for very short wavelengths but it is destabilizing otherwise . The magnetic field influence decreases the self gravitating destabilizing character but never suppressed it. This is due to the fact that the gravitational instability of sufficiently long waves will persist and the reason for that lies in the logarithmic singularity of the gravitational potential energy for infinite wavelengths.

X \ α	1	1.1	1.2	1.5	3
	σ*				
0.01	0.014528	0.014528	0.014528	0.014528	0.014528
0.06	0.074767	0.074767	0.074766	0.074764	0.074749
0.13	0.118961	0.118958	0.118955	0.118944	0.118854
0.19	0.153858	0.15385	0.15384	0.153808	0.153535

0.25	0.181693	0.181674	0.181653	0.18158	0.180967
0.31	0.203647	0.203611	0.203571	0.203431	0.202259
0.37	0.22045	0.220387	0.220318	0.220076	0.21805
0.43	0.232579	0.232478	0.232368	0.231977	0.228697
0.49	0.240348	0.240193	0.240022	0.239421	0.234347
0.55	0.243938	0.243707	0.243453	0.242556	0.23495
0.61	0.243413	0.243075	0.242706	0.241398	0.230215
0.67	0.238703	0.23822	0.237688	0.235808	0.219513
0.73	0.229574	0.228884	0.228126	0.225435	0.201617
0.79	0.215535	0.214548	0.213461	0.209589	0.174004
0.85	0.195644	0.194208	0.192623	0.186934	0.130186
0.91	0.167992	0.165814	0.163395	0.154575	***ω****
0.97	0.127855	0.124156	0.119974	0.103915	0.138438
1.03	***ω****	***ω****	***ω****	***ω****	0.205714
1.09	0.112383	0.118804	0.12546	0.146514	0.263027
1.15	0.172124	0.177414	0.103032	0.201574	0.316326
1.21	0.219629	0.0.224772	0.230274	0.248683	0.367858
1.27	0.261587	0.2669	0.272601	0.29179	0.418804
1.33	0.300299	0.305956	0.312034	0.332555	0.469919
1.39	0.336878	0.343007	0.349597	0.371875	0.521758
1.45	0.371969	0.378683	0.385903	0.410319	0.574775
1.51	0.405993	0.413403	0.421368	0.448295	0.629376
1.57	0.439258	0.447475	0.456304	0.486124	0.685937
1.63	0.472003	0.481146	0.490964	0.52408	0.74483
1.69	0.50443	0.514627	0.525569	0.562409	0.806424
1.75	0.536719	0.54811	0.56032	0.601349	0.871096
1.81	0.56904	0.581776	0.595412	0.641131	0.939233
1.87	0.601556	0.615803	0.631039	0.681989	1.01124
1.93	0.634431	0.650372	0.667394	0.724163	1.08753
1.99	0.667834	0.685667	0.704681	0.767903	1.16855
x_c	**1.041**	**1.036**	**1.031**	**1.014**	**0.912**

Table (1): Values of the temporal amplification σ* (or the oscillation frequency ω*) for Ho/ H_G = 0.1.

X \ α	1	1.1	1.2	1.5	3
	σ*				
0.01	0.014528	0.014528	0.014528	0.014528	0.014526
0.07	0.074713	0.074701	0.074688	0.074643	0.074261
0.13	0.118639	0.118568	0.118491	0.118218	0.115919
0.19	0.152887	0.152674	0.15244	0.151613	0.14454
0.25	0.179507	0.179025	0.178496	0.17662	0.160133
0.31	0.199455	0.198527	0.197504	0.193862	0.160318
0.37	0.213169	0.211543	0.209748	0.203299	0.138324
0.43	0.220727	0.218048	0.215076	0.204265	***ω****
0.49	0.22186	0.217606	0.21285	0.195178	0.148296
0.55	0.215852	0.209206	0.201676	0.172561	0.247027

0.61	0.201239	0.190775	0.178612	0.126806	0.340256
0.67	0.174927	0.157572	0.136046	*ω**	0.43501
0.73	0.128608	*ω**	*ω**	0.174784	0.533696
0.79	*ω**	0.112673	0.155959	0.255487	0.637595
0.85	0.160338	0.199261	0.234595	0.330185	0.747607
0.91	0.234252	0.270279	0.304892	0.40382	0.864485
0.97	0.300315	0.336889	0.372852	0.478444	0.988935
1.03	0.363837	0.402403	0.440787	0.555225	1.12166
1.09	0.426955	0.468405	0.509952	0.634982	1.26338
1.15	0.490848	0.535869	0.581189	0.71838	1.41485
1.21	0.556313	0.605502	0.655151	0.806009	1.57688
1.27	0.623967	0.677889	0.732405	0.898434	1.5703
1.33	0.694343	0.753556	0.813479	0.996213	1.93603
1.39	0.767932	0.833003	0.898884	1.09991	2.13502
1.45	0.845209	0.916724	0.989134	1.21012	2.3483
1.51	0.92665	1.00522	1.08476	1.32745	2.57697
1.57	1.01274	1.09899	1.18629	1.45254	2.82218
1.63	1.10397	1.19859	1.2943	1.58605	3.08519
1.69	1.20086	1.30455	1.40939	1.72871	3.36732
1.75	1.30395	1.41747	1.53217	1.88125	3.66999
1.81	1.41381	1.53795	1.66331	2.04449	3.9947
1.87	1.53103	1.66663	1.80349	2.21925	4.34308
1.93	1.65625	1.80421	1.95345	2.40643	4.71685
x_c	**0.784**	**0.756**	**0.73**	**0.66**	**0.441**

Table (2): Values of the temporal amplification σ* (or the oscillation frequency ω*) for Ho/ H_G = 0.5 .

x \ α	1	1.1	1.2	1.5	3
	*σ**				
0.01	0.014527	0.014527	0.014527	0.014526	0.014521
0.07	0.074544	0.074496	0.074444	0.074261	0.072713
0.13	0.117627	0.117341	0.117028	0.115919	0.106228
0.19	0.149811	0.148939	0.147977	0.14454	0.11186
0.25	0.172497	0.170483	0.168249	0.160133	*ω**
0.31	0.185748	0.181724	0.177213	0.160318	0.147716
0.37	0.188616	0.181145	0.172592	0.138324	0.263921
0.43	0.178689	0.165011	0.148592	0.628166	0.383624
0.49	0.150055	0.122634	*ω**	*ω**	0.512189
0.55	*ω**	*ω**	0.132055	0.247027	0.651323
0.61	0.134553	0.185776	0.229094	0.340256	0.80199
0.67	0.227679	0.273716	0.316546	0.43501	0.964971
0.73	0.310603	0.357936	0.403455	0.533696	1.14103
0.79	0.392106	0.442961	0.492672	0.637595	1.33099

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0.85	0.475038	0.530705	0.585634	0.747607	1.53574
0.91	0.560844	0.622298	0.683292	0.864485	1.75624
0.97	0.650473	0.718564	0.786404	0.988935	1.99357
1.03	0.744668	0.820203	0.895653	1.12166	2.24889
1.09	0.844085	0.927865	1.01169	1.26338	2.82346
1.15	0.949348	1.04219	1.13519	1.41485	2.81865
1.21	1.06108	1.16382	1.26682	1.57688	3.13594
1.27	1.1799	1.29343	1.4073	1.7503	3.47694
1.33	1.30648	1.43173	1.55738	1.93603	3.84335
1.39	1.44151	1.57945	1.71786	2.13502	4.23702
1.45	1.5857	1.73737	1.88957	2.3483	4.65994
1.51	1.73982	1.90633	2.07341	2.57697	5.11422
1.57	1.90467	2.0872	2.27033	2.82218	5.60215
1.63	2.08113	2.28091	2.48134	3.08519	6.12615
1.69	2.2701	2.48848	2.70751	3.36732	6.68883
1.75	2.47254	2.71094	2.95001	3.66999	7.29298
1.81	2.6895	2.94942	3.21004	3.9947	7.94157
1.87	2.92206	3.20513	3.4889	4.34308	8.6378
1.93	3.17139	3.47934	3.788	4.71685	9.38507
1.99	3.43873	3.7734	4.10879	5.11783	10.187
x_c	**0.5675**	**0.537**	**0.5095**	**0.441**	**0.2615**

Table (3): Values of the oscillation frequency ω* for Ho/ H_G =1.0

x \ α	1	1.1	1.2	1.5	3
			*σ**		
0.01	0.014521	0.014519	0.014518	0.014512	0.014462
0.07	0.072713	0.072273	0.071789	0.070057	0.053483
0.13	0.106228	0.103351	0.100106	0.087727	*ω**
0.19	0.11186	0.100829	*ω**	*ω**	0.277244
0.25	*ω**	*ω**	0.097945	0.183241	0.483162
0.31	0.147716	0.18742	0.222934	0.317842	0.727132
0.37	0.263921	0.307433	0.348918	0.466513	1.00833
0.43	0.383624	0.435292	0.485614	0.631683	1.32629
0.49	0.512189	0.574138	0.536091	0.814195	1.68108
0.55	0.651323	0.725202	0.79831	1.01467	2.07325
0.61	0.80199	0.889306	0.976015	1.23378	2.50379
0.67	0.964971	1.0672	1.16895	1.47232	2.97408
0.73	1.14103	1.25966	1.37792	1.73123	3.48587
0.79	1.33099	1.46757	1.60386	2.01161	4.04125

0.85	1.53574	1.69186	1.84777	2.31469	4.64263
0.91	1.75624	1.93359	2.1108	2.64187	5.29273
0.97	1.99357	2.19392	2.3942	2.9947	5.9946
1.03	2.24889	2.47413	2.69935	3.37488	6.75161
1.09	2.52346	2.77559	3.02774	3.78427	7.56743
1.15	2.81865	3.09981	3.38102	4.22489	8.44609
1.21	3.13594	3.4484	3.76095	4.69895	9.39193
1.27	3.47694	3.82312	4.16942	5.2088	10.4097
1.33	3.84335	4.22585	4.60849	5.75701	11.5044
1.39	4.23702	4.65861	5.08037	6.34632	12.6816
1.45	4.65994	5.12359	5.58742	6.97967	13.9471
1.51	5.11422	5.62311	6.13219	7.66025	15.3072
1.57	5.60215	6.15966	6.71739	8.39144	16.7687
1.63	6.12615	6.73594	7.34595	9.17688	18.3389
1.69	6.68883	7.35478	8.02097	10.0205	20.0256
1.75	7.29298	8.01927	8.74581	10.9264	21.837
1.81	7.94157	8.73268	9.52402	11.899	23.782
1.87	8.6378	9.4985	10.3594	12.9432	25.8703
1.93	9.38507	10.3205	11.2561	14.0641	28.1119
1.99	10.187	11.2026	12.2185	15.267	30.5177
x_c	**0.2615**	**0.2415**	**0.2245**	**0.185**	**0.0985**

Table (4): Values of the temporal amplification σ* (or the oscillation frequency ω*) for Ho/ H_G = 5.0

x \ α	1	1.1	1.2	1.5	3
			σ*		
0.01	0.014447	0.014429	0.014411	0.014344	0.013777
0.07	0.047118	0.038889	0.02715	0.044642	0.157299
0.13	0.133196	0.156331	0.178257	0.240024	0.522409
0.19	0.316938	0.355694	0.393788	0.505587	1.04572
0.25	0.544018	0.60419	0.663864	0.840956	1.71113
0.31	0.813933	0.900185	0.986032	1.24198	2.50893
0.37	1.12545	1.24213	1.35846	1.70613	3.43361
0.43	1.47797	1.62927	1.78029	2.23222	4.48265
0.49	1.87151	2.06162	2.2515	2.82018	5.65579
0.55	2.30666	2.53981	2.77276	3.47082	6.95461
0.61	2.78451	3.06502	3.34537	4.18575	8.38224
0.67	3.30659	3.63893	3.97113	4.9672	9.94317
0.73	3.87483	4.26365	4.65236	5.81808	11.6431
0.79	4.49155	4.94173	5.39183	6.74181	13.489
0.85	5.1594	5.6761	6.19274	7.74241	15.4888

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0.91	5.88144	6.4701	7.05872	8.8244	17.6515
0.97	6.66103	7.32743	7.99381	9.99285	19.9873
1.03	7.50192	8.25221	9.0025	11.2533	22.5072
1.09	8.40818	9.24893	10.0897	12.612	25.2237
1.15	9.38428	10.3225	11.2607	14.0755	28.1499
1.21	10.4351	11.4782	12.5214	15.6511	31.3004
1.27	11.5657	12.7219	13.878	17.3466	34.6909
1.33	12.782	14.0596	15.3373	19.1706	38.3382
1.39	14.0898	15.4982	16.9066	21.132	42.2607
1.45	15.4959	17.0447	18.5937	23.2407	46.4777
1.51	17.007	18.707	20.407	25.5072	51.0105
1.57	18.6309	20.4932	22.3556	27.9429	55.8815
1.63	20.3756	22.4123	24.449	30.5596	61.1148
1.69	22.2496	24.4736	26.6978	33.3705	66.7364
1.75	24.2622	26.6875	29.1129	36.3894	72.774
1.81	26.4234	29.0648	31.7063	39.631	79.2572
1.87	28.7436	31.617	34.4905	43.1113	86.2178
1.93	31.2343	34.3567	37.4793	46.8473	93.6897
1.99	33.9074	37.2972	40.6871	50.857	101.709
x_c					

Table (5): Values of the temporal amplification σ* (or the oscillation frequency ω*) for Ho/ H_G = 10

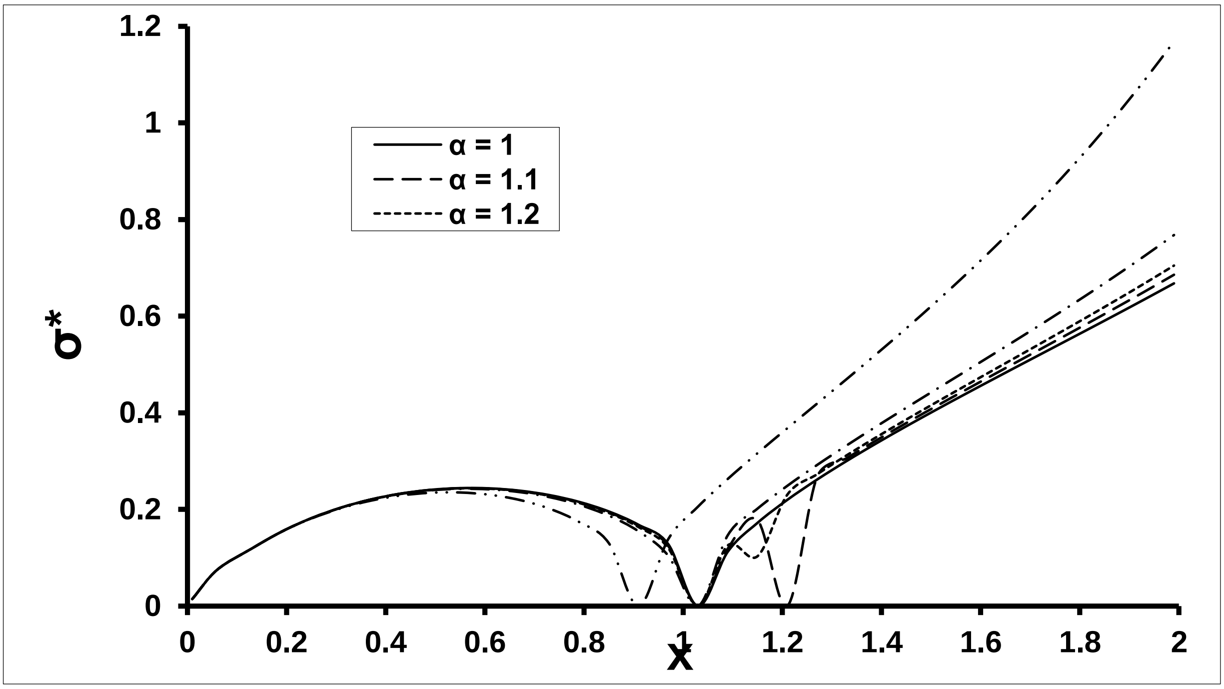

Figure (1) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for H_0/ H_G= 0.

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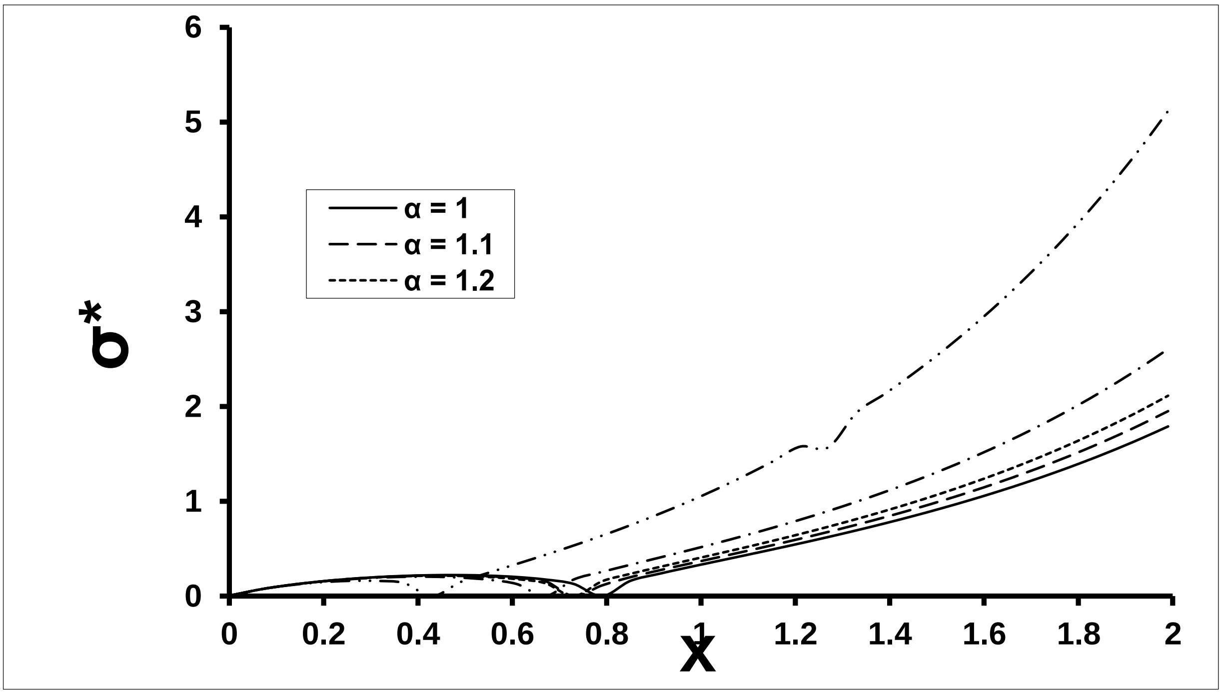

Figure (2) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for $H_0/H_G = 0.5$

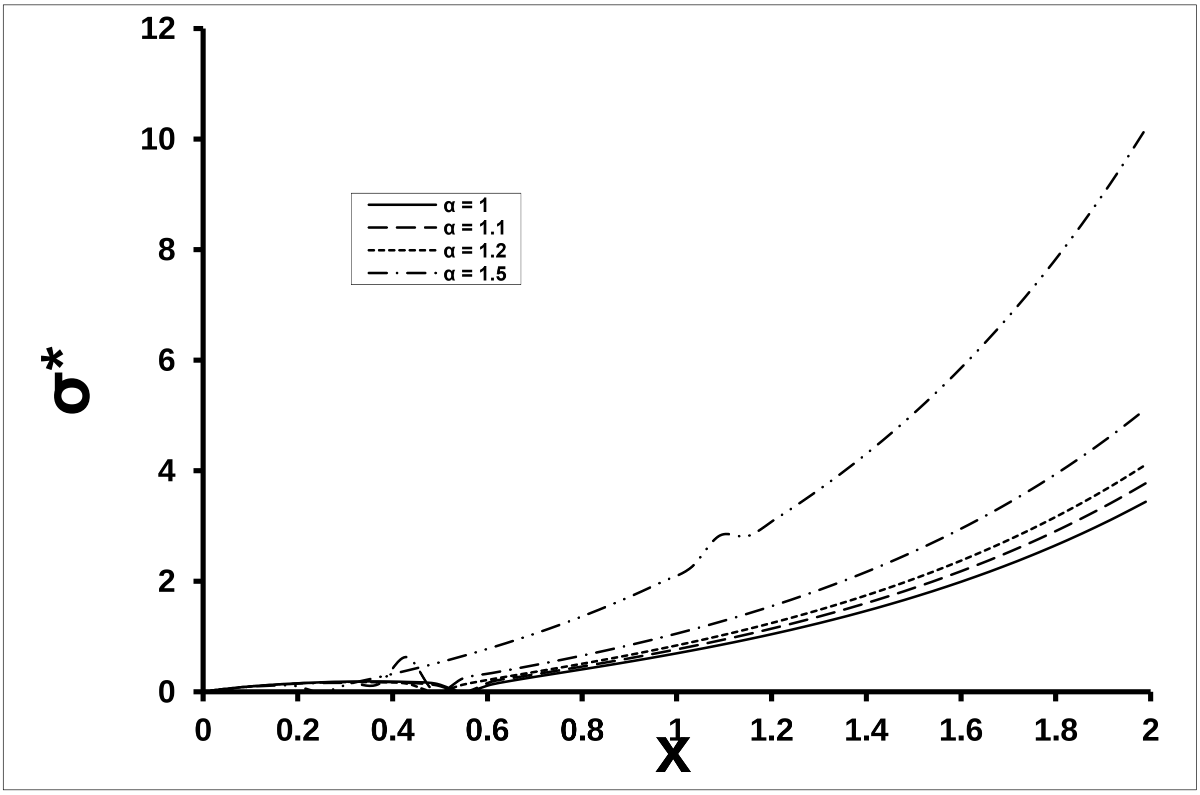

Figure (3) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for $H_0/H_G = 1.0$

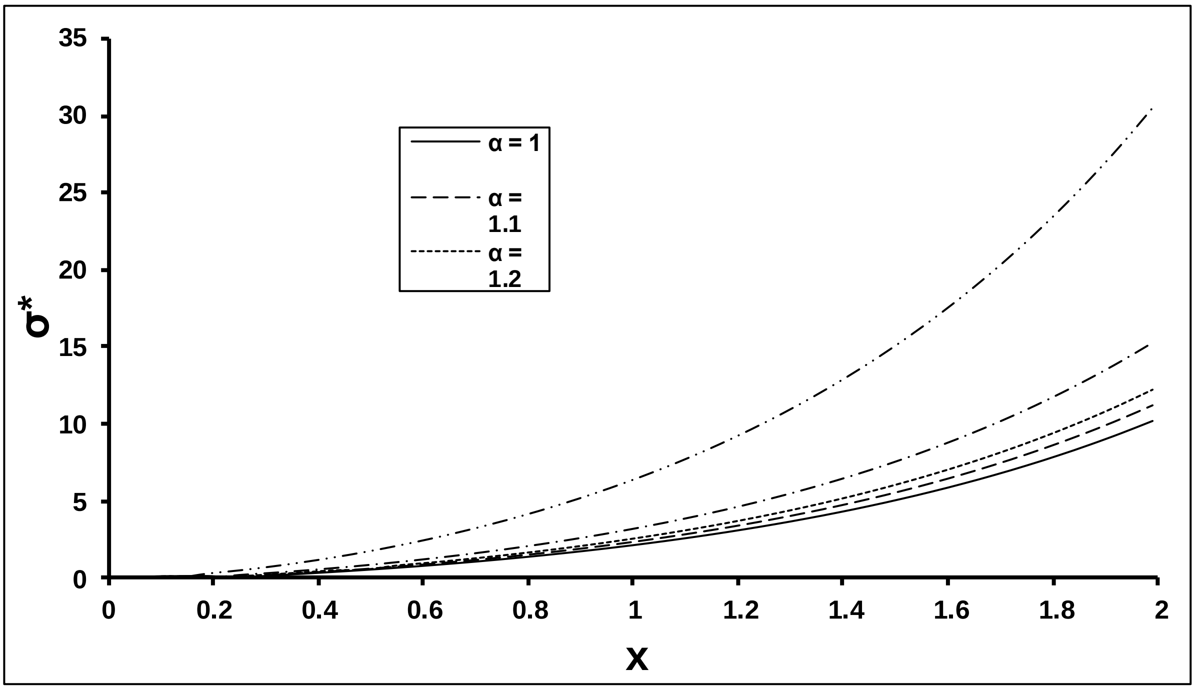

Figure (4) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for H_0/H_G= 3.0

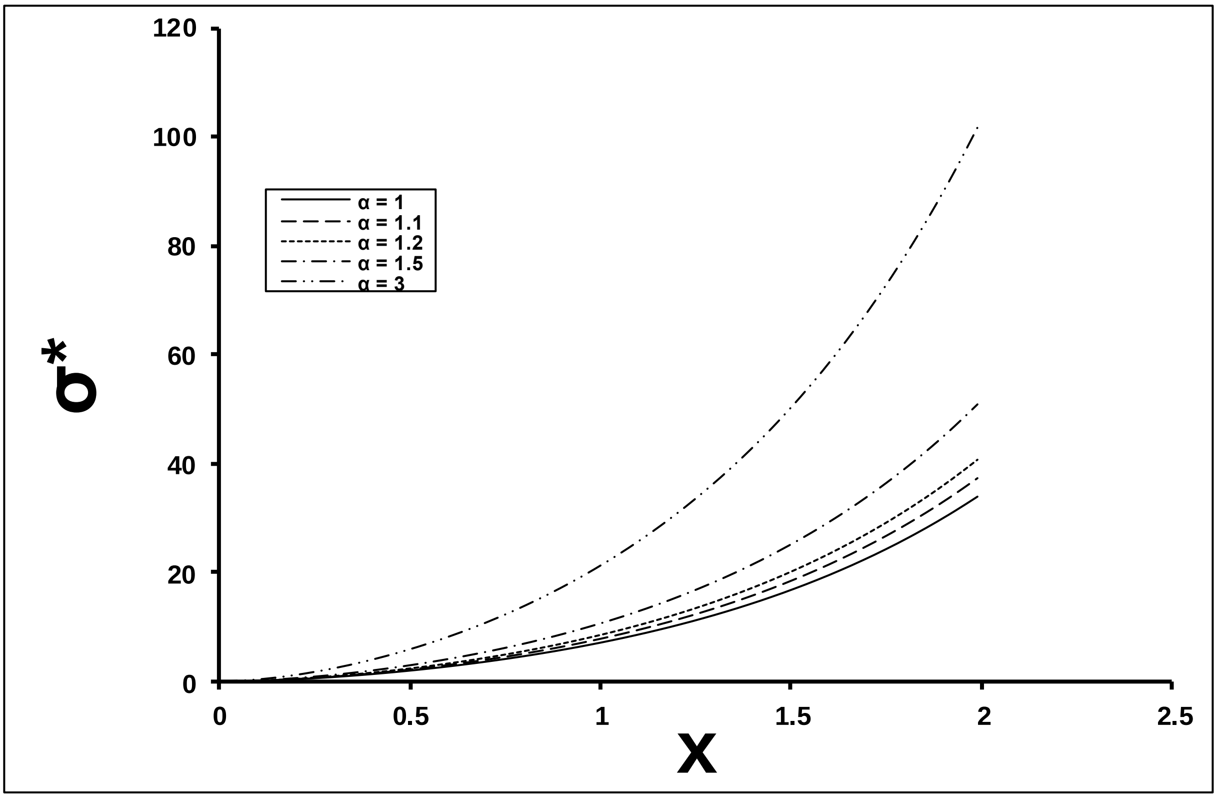

Figure (5) : Relation between the growth rate of oscillation σ^* and the dimensionless wavenumber x for H_0/H_G= 10

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